1. Photometry and spectroscopy of Nova Del 2013

60 p

a) From the light curve plot the Modified Julian Dates can be read with an error of about 0.5. The peak of the maximum brightness is very narrow and obviously placed at $\mathrm{MJD}_0 = 56\,520.5$ with a value of $m_0 = 4.5^{\mathrm{m}}$, therefore:

$$\mathrm{MJD}_0 = \boxed{56\,520.5 \pm 0.5} \qquad (2\,\mathrm{p})$$

$$m_0 = \boxed{4.5^{\mathrm{m}} \pm 0.05^{\mathrm{m}}} \qquad (1\,\mathrm{p})$$

b) The brightness values of 2^{m} and 3^{m} decline are $6.5^{\mathrm{m}} \pm 0.05^{\mathrm{m}}$ and $7.5^{\mathrm{m}} \pm 0.05^{\mathrm{m}}$, respectively. (2 p)

The corresponding Modified Julian Dates are MJD_2 and MJD_3. Because of the poorly defined slopes on the light curve around these dates, their acceptable error is larger than in other parts of the light curve, let say it is about 1^{d}, so:

$$\mathrm{MJD}_2 = \boxed{56\,531.5 \pm 1}, \quad \mathrm{MJD}_3 = \boxed{56\,543.5 \pm 1} \qquad (2\,\mathrm{p})$$

$$t_2 = \boxed{11^{\mathrm{d}} \pm 1^{\mathrm{d}}}, \quad t_3 = \boxed{23^{\mathrm{d}} \pm 1^{\mathrm{d}}} \qquad (2\,\mathrm{p})$$

c) The text of this part does not ask for calculating the individual errors of the formulae, but it is worth estimating them here, just for the sake of completeness. (Students won't do it.)

(a) The form of the function is

$$(1.1)\quad M = a + b \arctan \frac{c - \log t_2}{d}, \quad a = -7.92, b = -0.81, c = 1.32, d = 0.23,$$

so its derivative:

$$(1.2)\quad M' = -\frac{b}{d\log(10)\left[1 + \left(\frac{c-\log t_2}{d}\right)^2\right]} \frac{1}{t_2} \to \Delta M = -\frac{b}{d\log(10)\left[1 + \left(\frac{c-\log t_2}{d}\right)^2\right]} \frac{\Delta t_2}{t_2}$$

The value and its error calculated from the formulae above (error is not necessary):

$$M_0^{(a)} = \boxed{-8.63^{\mathrm{m}} \pm 0.06^{\mathrm{m}}} \qquad (1\,\mathrm{p})$$

(b) The form of the function is

$$(1.3)\quad M = a + b \log t_2, \quad a = -11.32, b = 2.55$$

so its derivative:

$$(1.4)\quad M' = \frac{b}{t_2 \log(10)} \to \Delta M = \frac{b}{\log(10)} \frac{\Delta t_2}{t_2}$$

The value and its error calculated from the formulae above (error is not necessary):

$$M_0^{(b)} = \boxed{-8.66^{\mathrm{m}} \pm 0.10^{\mathrm{m}}} \qquad (1\,\mathrm{p})$$

(c) The form of the function is

$$(1.5)\quad M = a + b \log t_3, \quad a = -11.99, b = 2.54$$

so its derivative:

$$(1.6)\quad M' = \frac{b}{t_3 \log(10)} \to \Delta M = \frac{b}{\log(10)} \frac{\Delta t_3}{t_3}$$

The value and its error calculated from the formulae above (error is not necessary):

$$M_0^{(c)} = \boxed{-8.53^{\mathrm{m}} \pm 0.05^{\mathrm{m}}} \qquad (1\,\mathrm{p})$$

The standard deviation of a dataset can be calculated as

$$(1.7)\quad \sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}},$$

where n is the number of data points, x_i is the i^{th} individual data value and $\bar{x}$ is their mean, $\bar{x} = (x_1 + x_2 + \ldots + x_n)/n$.

The mean and the standard deviation of the three absolute maximum brightness values:

$$M_0 = \boxed{-8.61^{\mathrm{m}} \pm 0.07^{\mathrm{m}}} \qquad (2\,\mathrm{p})$$

d) The color excess $E(B-V)$ is the difference between the observed color index of the star and the intrinsic color index predicted from its spectral type:

$$(1.8)\quad E(B-V) = (B-V) - (B-V)_0 = A_B - A_V$$

The total extinction is quantified by A_V (at 5550 Å). The ratio of total-to-selective extinction:

$$(1.9)\quad R = \frac{A_V}{E(B-V)} \to A_V = RE(B-V),\ \text{where } R = 3.1 \qquad (2\,\mathrm{p})$$

With the given value and error of $E(B-V)$:

$$A_V = 3.1 \times E(B-V) = 3.1 \times (0.184^{\mathrm{m}} \pm 0.035^{\mathrm{m}}) = \boxed{0.57^{\mathrm{m}} \pm 0.11^{\mathrm{m}}} \qquad (2\,\mathrm{p})$$

e) According to the formula for the distance modulus:

$$(1.10)\quad m_V - M_V = -5 + 5\log d + A_V \to \qquad (1\,\mathrm{p})$$

$$(1.11)\quad \log d = \frac{m_V - M_V + 5 - A_V}{5} \to$$

$$(1.12)\quad d = 10^{(m_V - M_V + 5 - A_V)/5}, \qquad (2\,\mathrm{p})$$

where the distance d is in parsecs.

Since $\Delta a^x/\Delta x = a^x \ln a$, therefore the error of the distance d:

$$\Delta d = 10^{(m_V - M_V + 5 - A_V)/5} \times \ln 10 \times \Delta((m_V - M_V + 5 - A_V)/5) \qquad (2\,\mathrm{p})$$

The error of $(m_V - M_V + 5 - A_V)/5$ can be estimated with the sum of the errors of m_V, M_V and A_V, so:

$$\Delta((m_V - M_V + 5 - A_V)/5) \approx 0.05 \qquad (2\,\mathrm{p})$$

With the data:

$$d = 3220\,\mathrm{pc} \text{ and } \Delta d = 338\,\mathrm{pc}, \qquad (2\,\mathrm{p})$$

so the distance to the nova:

$$d \approx \boxed{3.2 \pm 0.3\,\mathrm{kpc}} \qquad (2\,\mathrm{p})$$

f) The well known Doppler formula between the wavelength displacement and radial velocity:

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0} = \frac{v_\mathrm{r}}{c} \to v_\mathrm{r} = \frac{\Delta\lambda}{\lambda_0} c, \tag{1.13}$$ (1 p)

where λ is the measured wavelength of the line feature, λ_0 is the rest wavelength of the line, c is the speed of light, and v_r is the radial velocity to be calculated.

The wavelength of the P Cygni absorption peak should be extracted from the figure with an error of about 1 Å. The main part of this error is coming from the "definition" of the peak position of the Gaussian-like profile. This will result in inaccuracy of about $\Delta v_\mathrm{r} = \pm 50\,\mathrm{km\,s^{-1}}$ in radial velocities. (1 p)

The wavelengths and radial velocities should be something like these:

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MJD       WL   RV
56518.986 6527 -1636
56519.813 6531 -1454
56520.843 6534 -1317
56521.835 6537 -1179
56522.829 6542  -951
56523.827 6544  -860
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6 points for the wavelength values and 6 points for the radial velocity values. (12 p)

Radial velocities within the range of $\pm 50\,\mathrm{km\,s^{-1}}$ of the RV values listed in the table should be given full marks, but velocities in the range of $\pm 100\,\mathrm{km\,s^{-1}}$ are still acceptable with half marks.

g) See the attached figure as an example for the acceptable solution. The plotted data are taken from the table above. For the sake of simplicity the absolute values of the radial velocities have been used for making the graph. (6 p)

h) It is obvious from the plot, that the radial velocities lie along a straight line.

To estimate the size of the expanding envelope we need to calculate the area below the $t - v_\mathrm{r}$ graph between the first and last date.

Hence the graph is a straight line, this is very simple: we have to determine the area of the hatched region which is a trapezoid.

If the two bases and the height of the trapezoid are a, b, and m, respectively, then the area of the trapezoid is:

$$T = \frac{a+b}{2} m \tag{1.14}$$ (3 p)

In our case $a = v_{\mathrm{r}_1}$, $b = v_{\mathrm{r}_6}$, and $m = t_6 - t_1$. (1 p)

We could use the fitted line (dashed) as the upper side of the trapezoid, but this would be a bit complicated – because of the difficulties of the fitting process –, and not necessary at all. Instead of this we use the line connecting the first and last radial velocity points as this runs very close to the fitted line.

The result: $R \approx \boxed{3.5\,\mathrm{AU}}$ (3 p)

i) The apparent angular diameter of the spherical envelope seen from the Earth:

$$\vartheta = 2 \times \arctan\left(\frac{R}{d}\right) \approx 2\,\frac{R}{d} \tag{1.15}$$ (2 p)

Using the values of $d \approx 3.2\,\mathrm{kpc}$ and $R \approx 3.5\,\mathrm{AU}$, 5 days after the discovery the angular diameter of the envelope is:

$$\vartheta = \boxed{0.0022''} = \boxed{2.2\,\mathrm{mas}} \qquad (2\,\mathrm{p})$$

A less formal solution:

- By definition a parsec (1 pc) is the distance from the Sun to an astronomical object that has a parallax angle of one arcsecond, i.e. it represents the distance at which the radius of Earth's orbit (1 AU) subtends an angle of one arcsecond.
- Because of the very small angles the distance is a linear function of the parallax. This means that the radius $R \approx 3.5\,\mathrm{AU}$ of the envelope subtends one arcsecond viewing from a distance of $d \approx 3.5\,\mathrm{pc}$, and one milliarcsecond from a distance of $1000d \approx 3500\,\mathrm{pc} = 3.5\,\mathrm{kpc}$.
- Since this value is close to the distance of Nova Del 2013 determined earlier, we can conclude that the apparent angular diameter of the spherical shape envelope 5 days after the discovery was about 2 milliarcseconds.